NASA/CR- -1998- 208200

IN-27-CR
151416

Heat Conduction in Ceramic Coatings: Relationship between Microstructure and Effective Thermal Conductivity

Technical report for Task 2 (Second Year)

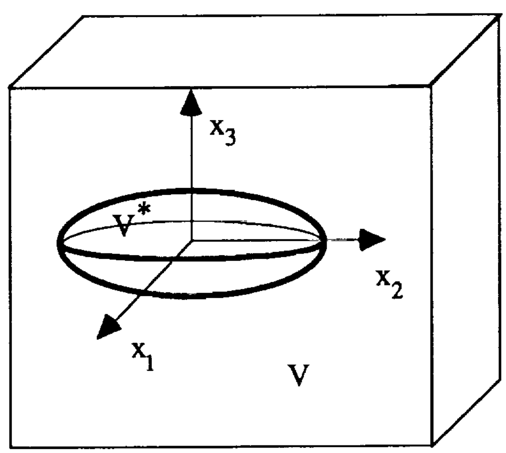
(period of performance: February 7, 1998 - May 6, 1998)

Contract Number: NAS3-97002

Analysis of the effective thermal conductivity of ceramic coatings and its relation to the microstructure continued. Results (obtained in Task 1) for the three-dimensional problem of heat conduction in a solid containing an inclusion (or, in particular, cavity - thermal insulator) of the ellipsoidal shape, were further advanced in the following two directions:

- closed form expressions of $\boldsymbol{H}$ tensor have been derived for special cases of ellipsoidal cavity geometry: spheroid, crack-like spheroidal cavity and needle shaped spheroidal cavity;
- these results for one cavity have been incorporated to contrast heat energy potential for a solid with many spheroidal cavities (in the approximation of non-interacting defects).

This problem constitutes a basic building block for further analyses, since the ellipsoidal shape covers a variety of practically important pore geometries.



The problem is formulated as the determination of the change in thermal conductivity due to inclusion. Namely:

$$\Delta \boldsymbol{Q} = \boldsymbol{H} \cdot \boldsymbol{G} \tag{1}$$

where ΔQ is the heat flux change per reference volume V, G is the far-field temperature gradient and second rank tensor H is a function of the inclusion shape and the inclusion conductivity.

Mathematical considerations based on the analysis of the ellipsoidal shapes in the framework of Eshelby-type theory and on utilization of Green's function for the heat conduction problem in an unbounded medium, show that H has the following form:

$$\boldsymbol{H} = \frac{V^*}{V}(k_* - k_0)(A_1\boldsymbol{ll} + A_2\boldsymbol{mm} + A_3\boldsymbol{nn}) \qquad (2)$$

where $V^* = \frac{4\pi}{3}a_1a_2a_3$ is the volume of the ellipsoidal inclusion with semi-axes a_1, a_2, a_3 with unit vectors $\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}$; k_* and k_0 are conductivities of the inclusion and of the matrix, correspondingly. Coefficients A_1, A_2, A_3 are given in terms of elliptic integrals (see Report for Task 1).

In the case when the inclusion is a *spheroid* $(a_1 = a_2 \equiv a)$, tensor H takes the form

$$\boldsymbol{H} = \frac{V^*}{V}(k_* - k_0)\left\{\left[1 + \frac{k_* - k_0}{k_0} f_0(\gamma)\right]^{-1}(\boldsymbol{I} - \boldsymbol{nn}) + \left[1 + \frac{k_* - k_0}{k_0}(1 - 2f_0(\gamma))\right]^{-1}\boldsymbol{nn}\right\} \qquad (3)$$

or, in components:

$$H_{ij} = \frac{V^*}{V}(k_* - k_0)\left\{\left[1 + \frac{k_* - k_0}{k_0} f_0(\gamma)\right]^{-1}(\delta_{ij} - n_in_j) + \left[1 + \frac{k_* - k_0}{k_0}(1 - 2f_0(\gamma))\right]^{-1} n_in_j\right\} \qquad (4)$$

where it is denoted:

$\gamma = a/a_3$ - aspect ratio of the spheroidal inclusion,

$\boldsymbol{n} = n_1\boldsymbol{e}_1 + n_2\boldsymbol{e}_2 + n_3\boldsymbol{e}_3$ - unit vector along the axis of symmetry of spheroid,

$\boldsymbol{I} = \boldsymbol{e}_1\boldsymbol{e}_1 + \boldsymbol{e}_2\boldsymbol{e}_2 + \boldsymbol{e}_3\boldsymbol{e}_3$ - unit second rank tensor,

$f_0(\gamma) = \dfrac{1 - g(\gamma)}{2(1-\gamma^2)}$, $\qquad g(\gamma) = \dfrac{\gamma^2}{\sqrt{\gamma^2 - 1}}\arctan\sqrt{\gamma^2 - 1}$ (for oblate shape, $\gamma > 1$),

$g(\gamma) = \dfrac{\gamma^2}{2\sqrt{1-\gamma^2}}\ln\dfrac{1 + \sqrt{1-\gamma^2}}{1 - \sqrt{1-\gamma^2}}$ (for prolate shape, $\gamma < 1$).

• In the case of *spheroidal cavity* (insulator, $k_* = 0$) tensor $\boldsymbol{H}$ is as follows:

$$\boldsymbol{H} = -\frac{V^*}{V} k_0 \left\{ \frac{1}{1+f_0(\gamma)} (\boldsymbol{I} - \boldsymbol{nn}) + \frac{1}{2f_0(\gamma)} \boldsymbol{nn} \right\} \tag{5}$$

• In the case of *thin spheroidal cavity* ($\gamma >> 1$):

$$\boldsymbol{H} = -\frac{V^*}{V} k_0 \left\{ \frac{1}{1+\pi/(4\gamma)} (\boldsymbol{I} - \boldsymbol{nn}) + \frac{2\gamma}{\pi} \boldsymbol{nn} \right\} \tag{6}$$

• In the limit of a *circular crack*:

$$\boldsymbol{H} = -\frac{8a^3}{3V} k_0 \boldsymbol{nn} \tag{7}$$

• In the case of *needle-shaped spheroidal cavity* ($\gamma << 1$):

$$\boldsymbol{H} = -\frac{V^*}{V} k_0 \left\{ \left[1 - \frac{1}{2}\left(1 + \gamma^2 - \gamma^2 \ln\frac{2}{\gamma} \right) \right]^{-1} (\boldsymbol{I} - \boldsymbol{nn}) + \left(1 + \gamma^2 - \gamma^2 \ln\frac{2}{\gamma} \right) \boldsymbol{nn} \right\} \tag{8}$$

In the approximation of non-interacting cavities (each cavity experiences the influence of the same far-field temperature gradient $\boldsymbol{G}$ unperturbed by the presence of other cavities), the *heat energy potential* $\Delta\Omega$ for a solid with many cavities is obtained as follows (in terms of derived tensors $\boldsymbol{H}^{(i)}$ characterizing $i-$th cavity):

$$\Delta\Omega = \frac{1}{2} \boldsymbol{G} \cdot \left[\sum_i \boldsymbol{H}^{(i)} \right] \cdot \boldsymbol{G} \tag{9}$$

For example, in the case of *spherical* cavities we have:

$$\sum_i \boldsymbol{H}^{(i)} = -\frac{3}{2} k_0 \boldsymbol{I} \left[\frac{1}{V} \sum_i V^{*(i)} \right] = -\frac{3}{2} p k_0 \boldsymbol{I} \tag{10}$$

$$\Delta\Omega = -\frac{3}{4} p k_0 \boldsymbol{G} \cdot \boldsymbol{G} = -\frac{3}{4} p k_0 \left(G_1^2 + G_2^2 + G_3^2 \right) \tag{10a}$$

where parameter p is the conventional porosity.

In the case of *circular cracks* we have:

$$\sum_i \boldsymbol{H}^{(i)} = -\frac{8}{3} k_0 \left[\frac{1}{V} \sum_i \left(a^3 \boldsymbol{nn} \right)^{(i)} \right] = -\frac{8}{3} k_0 \boldsymbol{\alpha} \tag{11}$$

$$\Delta\Omega = -\frac{4}{3} k_0 \, \boldsymbol{G} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{G} \tag{11a}$$

where $\boldsymbol{\alpha}$ is the second rank crack density tensor (well known in problems of effective *elastic* properties of cracked media).

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
May 12, 1998

3. REPORT TYPE AND DATES COVERED
Task 2 Report 2/7/98 - 5/6/98

4. TITLE AND SUBTITLE
CONTRACT NAS3-97002
"Heat Conduction in Ceramic Coatings: Relationship between Microstructure and Effective Thermal Conductivity"

5. FUNDING NUMBERS
WU-

6. AUTHOR(S)
Prof. Mark Kachanov

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Tufts University, Grants and Contracts Administration
Attn: Theodore M. Liszczak
Packard Hall, Room 103
Medford MA 02155

8. PERFORMING ORGANIZATION REPORT NUMBER
E-

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA CR-

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited
Subject Category

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Analysis of the effective thermal conductivity of ceramic coatings and its relation to the microstructure continued. Results (obtained in Task 1) for the three-dimensional problem of heat conduction in a solid containing an inclusion (or, in particular, cavity - thermal insulator) of the ellipsoidal shape, were further advanced in the following two directions:

- closed form expressions of $\boldsymbol{H}$ tensor have been derived for special cases of ellipsoidal cavity geometry: spheroid, crack-like spheroidal cavity and needle shaped spheroidal cavity;
- these results for one cavity have been incorporated to construct heat energy potential for a solid with many spheroidal cavities (in the approximation of non-interacting defects).

This problem constitutes a basic building block for further analyses.

14. SUBJECT TERMS

15. NUMBER OF PAGES 5

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102